Army Public School, Dagshai

Class – X , Winter Break Task on Mathematics

Aim of the task:- a) To prepare the students for Board Exams

b) To develop confidence and motivation towards the subject.

Solve the following questions in a thin notebook with well labeled figures wherever required.

1. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50^0$, then find $\angle AOB$.
2. In fig.1, AB is a 6 m high pole and CD is a ladder inclined at an angle of 60^0 to the horizontal and reaches up to a point D of pole. If AD = 2 **.** 54 m, find the length of the ladder. (use $\sqrt{3} = 1.73$)

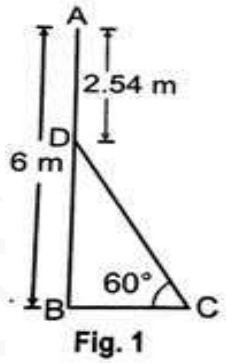


Fig. 1

3. Find the 9^{th} term from the end of the AP:- 5, 9, 13,…….185.
4. Cards marked with number 3, 4, 5,……., 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected card bears a perfect square number.
5. If x = 2/3 and x = – 3 are roots of $ax^2 + 7x + b = 0$, find the values of a and b.
6. Find the ratio in which y-axis divides the line segment joining the points A(5,– 6) and B(–1, –4). Also find the coordinates of the point of division.
7. In fig.2 a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12cm, 8cm and 10cm respectively, find the lengths of AD, BE and CF.

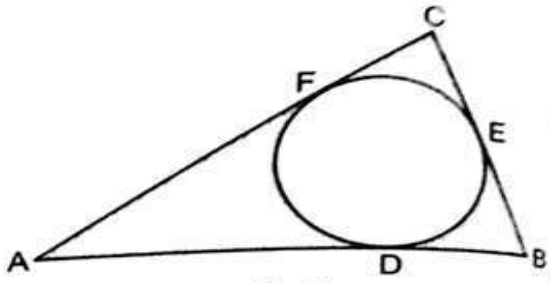


Fig. 2

8. The x-co-ordinate of a point P is twice its y-co-ordinate. If P is equidistant from Q(2, –5) and R(–3, 6). Find the co-ordinates of P.
9. How many term of the AP:- 18, 16, 14,……. be taken so that their sum is zero?
10. In fig.3, AP and BP are tangents to a circle with center O, such that AP = 5cm and $\angle APB = 60^0$. Find the length of chord AB.

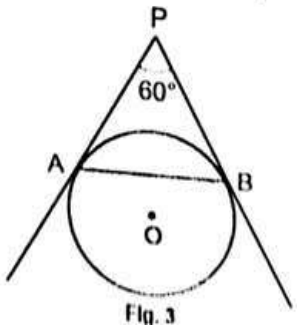


Fig. 3

11. In fig.4, ABCD is square of side 14cm. Semi-circles are drawn with each side of square as diameter. Find the area of shaded region. [use Π = 22 / 7]
12. In fig.5, a decorative block, made up of two solids – a cube and a hemisphere fixed on the top has a diameter of 3**.**5 cm. Find the total surface area of the block. [use Π = 22 / 7]

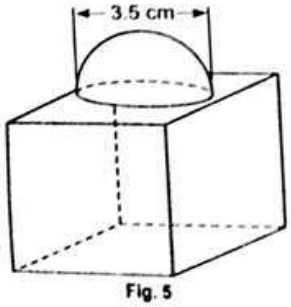


Fig. 5

13. In fig.6,Co-ordinates of point A in a ΔABC are A(0, –1). D and E respectively are the mid points of sides AB and AC and their coordinates are (1,0) and (0,1) respectively. If F is the mid-point of BC, find the areas of ΔABC and ΔDEF.

14. In fig.7, are shown two arcs PAQ and PBQ. Arc PAQ is a part of a circle with center O and radius OP while arc PBQ is a semi-circle drawn on PQ as diameter with center M. If $OP = PQ = 10\text{cm}$, show that area of shaded region is $25\ (\sqrt{3} - \pi/6) sq.cm.$

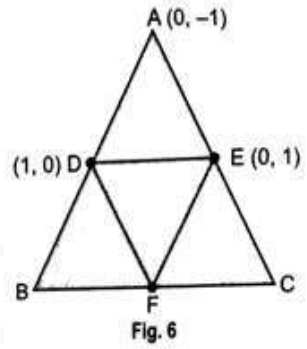


Fig. 6

15. If the sum of first 7 terms of an AP is 49 and that of its first 17 terms is 289, find the sum of first n terms of the AP.

16. Solve for x:-

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, -3/2$$

17. A well of diameter 4 m is dug 20 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment.

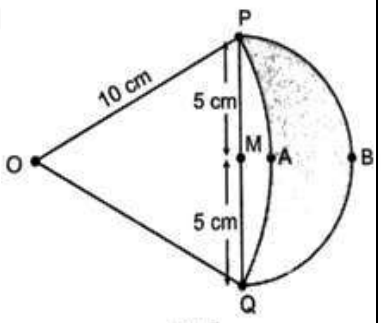


Fig. 7

18. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. [use Π = 22 / 7]

19. The angles of depression of top and bottom of a 50 m high building from the top of a tower are 45^0 and 60^0 respectively. Find the height of the tower and the horizontal distance between the tower and the building. (use $\sqrt{3} = 1.73$)

20. In a single throw of a pair of different dice, what is the probability of getting :-

i) A prime number on each dice?

ii) A total of 9 or 11?

21. A passenger, while boarding the plane, slipped from stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the flight got late by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250km/h than the usual speed. Find the usual speed of the plane.?

What value is depicted in this question?

22. Prove that the lengths of tangents drawn from an external point to a circle are equal.

23. Draw two concentric circles of radii 3 cm and 5 cm. Construct a tangent to the smaller circle from a point on the larger circle. Also measure its length.

24. In fig.8, O is the center of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.

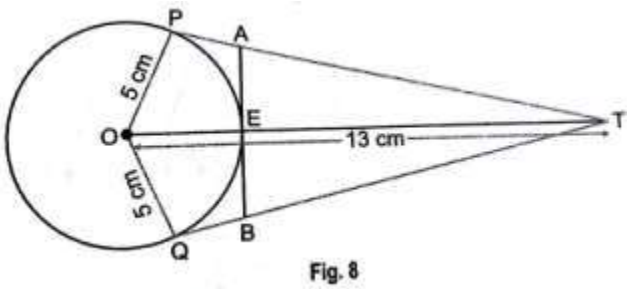


Fig. 8

25. Find x in terms of a, b and c:-

$$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$$

26. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45^0. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After two seconds, the angle of elevation of the bird from the same point is 30^0. Find the speed of flying of the bird. (use $\sqrt{3}=1.732$)

27. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief?

28. Prove that the area of a triangle with vertices (t, t – 2), (t + 2 , t + 2) and (t + 3, t) is independent of t.

29. A game of chance consists of spinning an arrow on a circular board, divided into 8 equal parts, which comes to rest pointing at one of the numbers 1, 2, 3,……,8. (Fig.9), which are equally likely outcomes. What is the probability that the arrow will point at:-

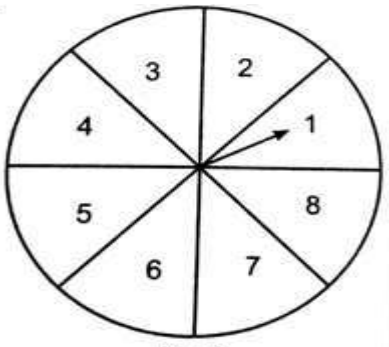


Fig. 9

i) An odd number, ii) A number greater than 3

iii) A number less than 9

30. An elastic belt is placed around a rim of pulley of radius 5 cm(fig.10). From one point C on the belt, the elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O.

Find the length of the belt that is still in contact with the pulley. Also find the shaded area.

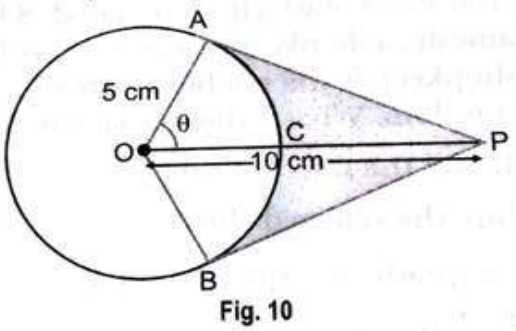


Fig. 10

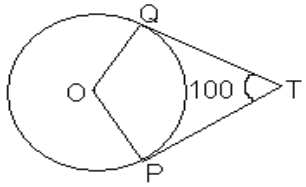
[Use $\Pi = 3.14$ and $\sqrt{3} = 1.73$]

31. A bucket open at top is in the form of a frustum of a cone with a capacity of $12308.8\ cm^3$. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and area of metal sheet used in making the bucket. [Use $\Pi = 3.14$]

32. Write the condition to be satisfied by q so that a rational number $\frac{p}{q}$ has a terminating decimal expansion.

33. For what value of 'a', the following pair of equations will have a unique solution?
4x + 3y = 3 and 8x + ay =5

34. Two tangents TP and TQ are drawn from an external point T to a circle with centre O, as drawn in the fig. If they are inclined to each other at an angle of 100^0 then what is the value of $\angle POQ$.



35. A bag contains 20 balls of different colours. The probability of drawing a green ball is $\frac{2}{5}$. How many green balls are there in the bag?

36. Show that: $\sin(50^0 + \theta) - \cos(40^0 - \theta) = 0$.

37. The perimeters of two similar triangles ΔABC and ΔDEF are 36cm and 24cm respectively. If DE =10cm, find AB.

38. Give an example of polynomials f(x), g(x), q(x) and r(x) satisfying

f(x) = g(x). q(x) + r(x) where deg q(x) = deg r(x)

39. What is the distance between two parallel tangents of a circle of radius 5cm?

40. The angles of a triangle are in A.P, the least being half the greatest. Find the angles.

41. The median can graphically be found from which of the following types of graphs:

(a) Ogive (b) histogram

(c) Frequency curve (d) none of these

42. If the sum of first n terms of an AP is $3n^2 - 2n$, find the AP and its 19^{th} term.

43. Without using tables, find the value of

$$\frac{\cos^2 20 + \cos^2 70}{\sec^2 50 - \tan^2 50} + \tan 48 \tan 42 - 4 \sin^2 45$$

44. If 3 tan A = 4, find the value of $\frac{5 \sin A - 3 \cos A}{5 \sin A + 2 \cos A}$

45. Find the value of p for which the points (-5,1), (1,p) and (4,-2) are collinear.

46. From a pack of 52 cards King , Queen, Jack and Ace of hearts are removed and the remaining cards are well shuffled. A card is drawn at random; find the probability that the card drawn is

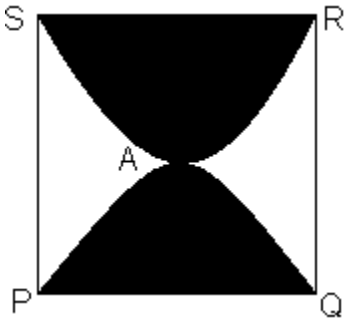
i) neither a red nor a queen.

ii) either a face card or an ace

47. A vertical stick of length 6m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28m long. Find the height of the tower.

48. Prove that √7 is an irrational number.

49. PQRS is a square land of side 28m. Two semicircular grass covered portions are to be made on two of its opposite sides as shown in the figure. How much area will be left uncovered? (Take $\pi = \frac{22}{7}$)



50. Find the zeroes of the quadratic polynomial $x^2 - 15x + 36$ and verify the relationship between the zeroes and the coefficients.

51. For what value of 'k', the following pair of linear equations has infinitely many solutions?

$10x + 5y - (k-5) = 0$

$20x + 10y - k = 0$

52. Find the solution of the pair of equations:

$$\frac{x}{2} + \frac{2y}{3} = -1 \quad and \quad x - \frac{y}{3} = 3$$

53. In a garden there are 30 mango trees in the first row, 27 in the second, 24 in the third and so on. There are 6 trees in the last row. How many rows are there of mango trees?

54. Prove that: $\frac{1}{\operatorname{cosec}A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec}A + \cot A}$

55. Construct a triangle ABC with AB = 5.5cm, BC = 6cm and $\angle ABC = 45^0$. Then construct a triangle whose sides are $\frac{1}{2}$ times the corresponding sides of ΔABC.

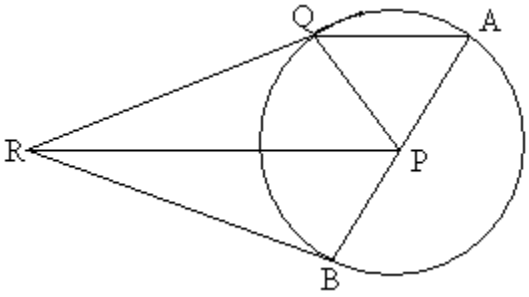
56. Find the sum of all multiples of 13 lying between 100 and 999.

57. Determine A.P whose third term is 16 and when 5^{th} term is subtracted from 7^{th} term, we get 12.

58. Find the point on the x-axis which is equidistant from the points (-2,5) and (2,-3). Hence find the area of the triangle so formed.

59. Prove that the centroid of triangle ABC whose vertices $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3, y_3)$ are given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

60. QR is a tangent at Q to a circle whose centre is P. PR is parallel to AQ where AQ is a Chord through A, the end point of diameter AB. Prove that BR is tangent at B.



61. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above theorem do the following:

ABC is a triangle. XY ‖ BC is a line segment intersecting AB in X and AC in Y and divides ΔABC into two parts equal in area. Prove that $\frac{BX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$.

62. Prove that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of other two sides. Use the theorem, in the following:- If ABC is an equilateral triangle with $AD \perp BC$, then $AD^2 = 3DC^2$.

63. Form a pair of linear equations in two variables using the following information and solve it graphically.

Yash scored 40 marks in a test, receiving 3 marks for each right answer and loosing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each wrong answer, then Yash would have scored 50 marks. How many questions were there in the test? (Assume that no question is left unanswered)

64. The angle of elevation of a jet plane from a point A on the ground is 60◦. After a flight of 15 seconds, the angle of elevation changes to 30◦. If the jet plane is flying at a constant height of 1500√3 m**,** find the speed of the jet plane.

65. A container made up of a metal sheet is in the form of a frustum of a cone of height 16cm with radii of its lower and upper ends as 8cm and 20cm respectively. Find the cost of milk which can completely fill the container @ of Rs.15 per litre and the cost of metal sheet used,, if it costs Rs.5 per 100cm^2. (Take π=3.14)

66. Right circular cone having diameter 12cm and height 15cm is full of ice-cream. The ice-cream is to be filled in cones of height 12cm and diameter 6cm having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

67. If the median of the distribution given below is 27, find the values of x and y.

Marks	**. of Students**
0-10	5
10-20	x
20-30	20
30-40	14
40-50	y
50-60	4
Total	68

Solve the following questions

Q1 If α & β are the roots of the eqn $x^2 - x - 90 = 0$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

Q2 Find the roots of :- (a) $4x^2 - 4px + (p^2 - q^2) = 0$ (b) $abx^2 + (b^2 - ac)x - bc = 0$ (c) $3x^2 - 2\sqrt{6}x + 2 = 0$.

Q3 Solve the following : (a) $9x^2 - 15x + 6 = 0$ by completing the square (b) $36x^2 - 12ax + (a^2 - b^2) = 0$ by quadratic formula.

Q4 Solve for x :- (a) $\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{1}{a+b+x}$ (b) $\left(\frac{2x}{x-5}\right)^2 + \frac{10x}{x-5} - 24 = 0$ (c) $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

Q5 For what value of 'k' does $(k-12)x^2 + 2(k-12)x + 2 = 0$ have equal roots ?

Q6 If roots of a equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ are real & equal, then prove $2b = a + c$.

Q7 If one root of $x^2 - 5x + 6k = 0$ is reciprocal of other, find the value of k. Also, find roots.

Q8 If $(1+m^2)n^2x^2 + 2mncx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1+m^2)$.

Q9 A motor boat with speed in still water as 18 km/hr takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Q10 Two water taps together can fill a tank in $9\frac{3}{8}$ hrs. The tap of larger diameter takes 10 hrs less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Q11 The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

Q12 For the same amount of work, an adult takes 6 hrs less than a boy. If together they complete the work in 15 hrs and 20 min. Find how much time will the boy alone take to complete the work.

Q13 How many 3-digit natural numbers are divisible by 7 ?

Q14 Which term of the AP :- 120, 116, 112, ... is its first negative term.

Q15 Find the middle term of the AP :- $-11, -7, -3, \ldots\ldots 45$.

Q16 If the sum of all the terms of an AP :- $1, 4, 7, 10, \ldots\ldots x$ is 287, find x.

Q17 If the sum of three numbers in AP is 21 and their product is 231, find the numbers.

Q18 In Nov 2009, the no. of visitors to a zoo increased daily by 20. If total of 12300 people visited the zoo in that month, find the no. of visitors on 1st Nov 2009.

Q19 Find the sum of all the three digit number which leave the remainder 2 when divided by 3.

Q20 Razia donated ₹ 70,0000 to a school from her savings for giving 7 cash prizes to students for their academic performances. If the worth of each prize is ₹ 20000 less than the worth of the preceding prize, find worth each prize. What value is depicted from this action ?

Q21 The sum of n, $2n$, $3n$ terms of an A.P are S_1, S_2, S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$

Q22 Find 'x', if the distance between the points $(x, -1)$ and $(3, -2)$ is $\sqrt{5}$.

Q23 Find the distance between the points $P\left(\frac{\sin\theta}{2}, 0\right)$ and $Q\left(0, \frac{\cos\theta}{2}\right)$

Q24 Find the value of k, if the point $P(0, 2)$ is equidistant from the point $A(3, k)$ and $B(k, 5)$

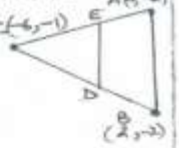
Q25 Prove that points $A(0, -1)$, $B(-2, 3)$, $C(6, 7)$ and $D(8, 3)$ are the vertices of rectangle ABCD.

Q26 In what ratio is the line segment joining the points $P(-2, -3)$ and $Q(3, 7)$ divided by the y-axis ?

Q27 Three consecutive vertices of ||gm ABCD are $A(1, 2)$, $B(1, 0)$ and $C(4, 0)$. Find the fourth vertex D.

Q28 In $\triangle ABC$, $G(2, 3)$ is a centroid. If A, B, C are the points $(1, 3)$, (k, b) and $(a, 1)$ resp. find 'a' and 'b'. Also find the length of side BC.

Q29 In the figure given along side, D and E are the mid-points of sides BC and AC resp. of $\triangle ABC$. Find the lengths of DE and AB. Prove that $DE = \frac{1}{2}AB$



Q30 If $A(4, -6)$, $B(3, -2)$ and $C(5, 2)$ are the vertices of $\triangle ABC$, then verify the fact that median of $\triangle ABC$ divides it into two $\triangle$s of equal area.

Q31 Four points $A(6, 3)$, $B(-3, 5)$, $C(4, -2)$ and $D(x, 3x)$ are given in such a way that :-
$\frac{ar.\triangle DBC}{ar.\triangle ABC} = \frac{1}{2}$. Find x.

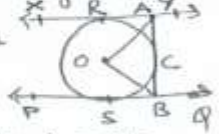
Q32 The angles of elevation of the top of a tower from two points at a distance of 'a' and 'b' from the base and in the same straight line with it are complementary. Prove that height of the tower is $\sqrt{ab}$.

Q33 From the I floor of Qutab Minar, which is at a height of 25 m from the level ground, a man observes the top of a building at an angle of elevation of 30° and the angle of depression of the base of the building is 60°. Calculate the Height of the building.

Q34 A man standing on a deck of a ship which is 10 m above the sea level, observes the angle of elevation of the top of the cloud as 30° & angle of depression of its reflection in the sea was found to be 60°. Find the height of the cloud & also the distance of the cloud from the ship.

Q35 A straight highways leads to the foot of a tower.
A man standing at the top of tower observers a car at an angle of depression 30°, which is approaching the foot of the tower with uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

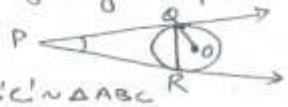
Q36 If XY and PQ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and PQ at B, prove that $\angle AOB = 90°$



Q37 Prove that the angle between the two tangents to a circle drawn from an external point, is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Q38 Prove that tangents drawn from an external point to a circle are equal.

Q39 PQ and PR are two tangents as shown. Prove $\angle QPR = 2\angle OQR$.
where 'O' is the centre of the circle.



Q40 Draw a $\triangle ABC$ with $BC = 6$ cm, $AB = 5$ cm, $\angle ABC = 60°$. Construct $\triangle A'B'C' \sim \triangle ABC$ such that sides of $\triangle A'B'C'$ are $\frac{3}{4}$ of the corresponding sides of $\triangle ABC$.

Q41 Solve Questions from Ex 12.2 - Q11, Pg-234-Exp-06, Ex 12.3 - Q6, Q9, Q10, Q15, Q16

Q42 Solve Questions from Ex 13.1 - Q5, Q9, Pg-249-Exp 7, Ex 13.3 - Q4, Q8, Q9, Ex 13.4 - Q4, Q5.

Q43 Solve Questions from Ex 15.1 - Q13, Q17, Q22, Q23, Q15.